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The non-synergy field of convection

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ABSTRACT

The "field synergy principle", a theory to approach best convection result proposed by Guo, Tao et al., has been developing since the end of last century. Besides the principle itself, there have been a lot of studies about the field synergy including computational approaches and experimental means to demonstrate and apply this principle. However, an opposite research direction: " non-synergy" – how can we obtain worst convection result – is also worth researching for the theory and practice. Until now detailed studies have hardly been published in the open literatures. In this paper, a basic theoretical non-synergy research is presented, some algebraically explicit analytical exact solutions for the governing partial differential equation set of two-dimensional laminar incompressible full field non-synergy are derived using the extraordinary methods promoted by the authors, for example, the method of separating variables with the addition and other hybrid method. The obtained solutions include the conditions with the heat source, the mass flow source or no any sources. The physical feature of various solutions are discussed and explained by the means of figures. Besides theoretical meaning, the solutions can be benchmark solutions to develop the computational heat transfer.

1. Non-synergy field principle

The convection is one of the very common processes in the earth. It occurs perhaps everywhere, for example, in the natural processes, in the production of the industry and the agriculture, and in the life cycles of all living beings. How to control the convection processes, for example, to promote or restrain them, is very important for human beings. In recent years, Guo, Tao et al. [1–3] proposed the field synergy principle with mathematics, the key issue is that the most effective convection occurs when the movement direction of all convection particles is completely perpendicular to the isothermals (2-D case) or the isothermal surfaces (3-D case). They confirmed this principle using many numerical and experimental studies [4–9], and have applied this principle to improve some heat transfer apparatuses and obtained excellent results.

Cai explained and derived again the field synergy principle with only simplest qualitative thinking, and gave several analytical exact solutions of full field synergy [10]. It is well-known that the analytical exact solutions have their own theoretical meaning, many analytical solutions played a key role in the early development of fluid mechanics and heat conduction [11,12]. Besides their theoretical meaning, analytical solutions can also be applied to check the accuracy, convergence and effectiveness of computation methods, and to improve their differencing schemes, grid generation techniques and so on. Therefore, the analytical solutions are very useful to the computational fluid dynamics and heat transfer.

As mentioned by Tao, Guo et al. [3,5], contrary to the field synergy, there is also the field non-synergy; in this case, the streamlines and the isothermals are all located at the same position, which is also very clear from a qualitative physical thinking. When all fluid particles flow along the isothermals, there should not be any convection effects [10]. If neglecting some other small heat transfer effects such as conduction, it approximates to an adiabatic case. Such conditions are valuable for the theory and many practical applications. Therefore, some analytical exact solutions for 2-D non-synergy will be derived to develop the field non-synergy principle and promote its practice.

2. Governing equation set and deriving analytical solutions

For a steady 2-D incompressible laminar flow with constant kinematic viscosity ν and thermal diffusivity a (neglecting gravity and dissipation heat), the governing equations can be presented as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial \upsilon}{\partial y}=G, \tag{1}$$

$$u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right), \tag{2}$$

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Nomenclature

a	thermal diffusivity, m^2/s
C_p	specific heat, J/(kg K)
c_i	arbitrary constant
$f(y)$	arbitrary function of y
G	mass rate, kg/s
$g(y)$	arbitrary function of y
p	pressure, Pa
q	heat source, W/m^2
u	velocity component in x direction, m/s
υ	velocity component in y direction, m/s
X	function of x
x	x coordinate
Y	function of y
y	y coordinate

Greek symbols

ν	kinematic viscosity, kg/(m s)
ρ	density, kg/m^3
θ	temperature, K

Subscripts

p	function for pressure
u	function for velocity component u
υ	function for velocity component υ
θ	function for temperature

$$u\frac{\partial \upsilon}{\partial x}+\upsilon\frac{\partial \upsilon}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 \upsilon}{\partial x^2}+\frac{\partial^2 \upsilon}{\partial y^2}\right), \quad (3)$$

$$u\frac{\partial \theta}{\partial x}+\upsilon\frac{\partial \theta}{\partial y}=a\left(\frac{\partial^2 \theta}{\partial x^2}+\frac{\partial^2 \theta}{\partial y^2}\right)+q. \quad (4)$$

Considering the condition of full field non-synergy, the following equation must be added for the full convection field:

$$\upsilon/u=-\frac{\partial\theta}{\partial x}\Big/\frac{\partial\theta}{\partial y}. \quad (5)$$

Commonly, q (heat source) and G (mass flow source) are given functions, then there are only four unknown variables in the five equations abovementioned, they are velocity components u and υ, pressure p and temperature θ; then the equation number is more than the number of the independent variables, it is not properly identified and generally unable to obtain solutions. Then q or G has to be an unknown variable to meet the number of equations. Actually, it is difficult to find a fully field non-synergy condition without any artificial measures. If some measures are adopted to control the field, such as adding appropriate heat source or mass flow source, the field non-synergy convection cases can be obtained. Therefore, q or G in the governing equation set has to be recognized as a control measure and an important variable.

The governing equation set (2)–(5) is composed of nonlinear simultaneous partial differential equations, which are not easy to be solved. In order to obtain algebraically explicit exact analytical solutions for understanding the results distinctly, the following methods have been adopted: The method of separating variables with addition proposed by Cai [13,14] is applied. It is assumed that the unknown solution can be expressed as $f(x,y) = X(x) + Y(y)$, instead of $f(x,y) = X(x)\cdot Y(y)$ in the common method of separating variables. Since the main motivation of deriving analytical solution here is to obtain some possible explicit analytical solutions to develop the non-synergy theory and promote numerical heat transfer (NHT), but not to find a specified solution for the given boundary conditions. Therefore, the boundary conditions are undetermined before deriving the explicit analytical solution and deduced from the solution afterward. It makes the derivation procedure easier. Actually, the basic solutions of incompressible fluid dynamics in early time did adopt such methods. The abovementioned approaches have been successfully applied to derive many meaningful algebraically explicit analytical solutions for the heat and mass transfer discipline [15–27].

In fact, all solutions given in this paper can be proven easily by substituting them into the governing equation set.

If the method of separating variables with addition is applied to all variables in Eqs. (1)–(5), the following equation set can be obtained.

$$u=X_u+Y_u, \quad (6)$$
$$\upsilon=X_\upsilon+Y_\upsilon, \quad (7)$$
$$p=X_p+Y_p, \quad (8)$$
$$\text{and}\quad \theta=X_\theta+Y_\theta, \quad (9)$$

Therefore, Eqs. (1)–(5) can be rewritten as:

$$X'_u+Y'_\upsilon=G(x,y), \quad (10)$$
$$(X_u+Y_u)X'_u+(X_\upsilon+Y_\upsilon)Y'_u=-X'_p/\rho+\nu(X''_u+Y''_u), \quad (11)$$
$$(X_u+Y_u)X'_\upsilon+(X_\upsilon+Y_\upsilon)Y'_\upsilon=-Y'_p/\rho+\nu(X''_\upsilon+Y''_\upsilon), \quad (12)$$
$$(X_u+Y_u)X'_\theta+(X_\upsilon+Y_\upsilon)Y'_\theta=a(X''_\theta+Y''_\theta)+q(x,y), \quad (13)$$
$$\text{and}\quad (X_u+Y_u)X'_\theta=-(X_\upsilon+Y_\upsilon)Y'_\theta. \quad (14)$$

Of course, a hybrid approach with both separating methods can be also applied for the equation set with multiple unknown variables.

3. Analytical full field non-synergy solution with heat source (I) – using the method of separating all variables with addition

To control the 2-D heat transfer field, a distributed heat source can easily be set in practice, for example, the radiation. Then, a simple non-synergy solution with a heat source only is derived firstly; after that, its simplified form with very clear physical meaning is given in the next paragraph.

For ease of derivation, in this work, we set $G = 0$.

For such case, Eq. (10) can be separated easily as:

$$X'_u=c_1=-Y'_\upsilon, \quad (15)$$
$$\text{then}\quad X_u=c_1x+c_2 \quad (16)$$
$$\text{and}\quad Y_\upsilon=c_3-c_1y \quad (17)$$

If $X_\upsilon = c_4$, then Eq. (11) can be separated as

$$c_1^2x+c_1c_2+X'_p/\rho=c_5=-c_1Y_u-(c_3+c_4)Y'_u+c_1yY'_u+\nu Y''_u. \quad (18)$$

The right-hand side of Eq. (18) can be analytically solved only when $c_1 = 0$. Since the aim of this paper is to find analytical exact solutions, $c_1 = 0$ is chosen further. Then from left-hand side and right-hand side of Eq. (18), following results can be obtained:

$$X_p=p_0+c_5\rho x, \quad (19)$$
$$\text{and}\quad Y_u=[\nu/(c_3+c_4)]^2\exp[(c_3+c_4)(y+c_6)/\nu]-c_5y/(c_3+c_4). \quad (20)$$

Since $X'_u = Y'_\upsilon = 0$, we can obtain $Y'_p = 0$ from Eq. (12). It means that Y_p is a constant. According to Eqs. (8) and (19), it can be regarded as zero:

$$Y_p=0. \quad (21)$$

Substituting above results (including $c_1 = 0$) into Eq. (14) yields:

$$\left\{[\nu/(c_3+c_4)]^2 \exp[(c_3+c_4)(y+c_6)/\nu] - c_5 y/(c_3+c_4) + c_2\right\}\frac{1}{Y'_\theta} = c_7 = -(c_3+c_4)\frac{1}{X'_\theta}. \quad (22)$$

The left-hand side of Eq. (22) can only be analytically solved when $c_2 = 0 = c_5$, we use such simplification further, then following two solutions are derived (since $c_3 + c_4$ always appears together in the final results, it is chosen c_3 on behalf of $c_3 + c_4$ in following equations)

$$Y_\theta = \nu^3 \exp[c_3(y+c_6)/\nu]/(c_7 c_3^3), \quad (23)$$

$$X_\theta = \theta_0 - c_3 x/c_7. \quad (24)$$

Substituting above solutions into Eq. (13) yields:

$$q = -a\nu g(y)/(c_3 c_7), \quad (25)$$

$$\text{where} \quad g(y) = \exp[c_3(y+c_6)/\nu]. \quad (26)$$

Combining Eqs. (10)–(14) with all previous results in this paragraph, the final solutions are

$$u = (\nu/c_3)^2 g(y), \quad (27)$$

$$\upsilon = c_3, \quad (28)$$

$$p = p_0, \quad (29)$$

$$\theta = \theta_0 - c_3 x/c_7 + (\nu/c_3)^3 \cdot g(y)/c_7 \quad (30)$$

$$q = -a\nu g(y)/(c_3 c_7). \quad (31)$$

where $g(y)$ has been given in Eq. (26). As mentioned before, the boundary conditions are determined after successfully deriving solutions. The conditions can be obtained by substituting the geometries of boundaries into the solutions. For example, if considering the boundary is a square with unit width, the boundary conditions of the solution of this paragraph could be:

$$y = 0, \; u = (\nu/c_3)^2 \exp(c_3 c_6/\nu),$$
$$\theta = \theta_0 - c_3 x/c_7 + (\nu/c_3)^3 \cdot \exp(c_3 c_6/\nu)/c_7;$$

and

$$y = 1, \; u = (\nu/c_3)^2 \exp[c_3(1+c_6)/\nu],$$
$$\theta = \theta_0 - c_3 x/c_7 + (\nu/c_3)^3 \cdot \exp[c_3(1+c_6)/\nu]/c_7;$$

in addition,

$$x = 0, \; u = (\nu/c_3)^2 \exp[c_3(y+c_6)/\nu],$$
$$\theta = \theta_0 + (\nu/c_3)^3 \cdot \exp[c_3(y+c_6)/\nu]/c_7,$$

$$x = 1, \; u = (\nu/c_3)^2 \exp[c_3(y+c_6)/\nu],$$
$$\theta = \theta_0 - c_3/c_7 + (\nu/c_3)^3 \cdot \exp[c_3(y+c_6)/\nu]/c_7.$$

In the whole field $\upsilon = c_3$ and $p = p_0$. The distribution of q can be recognized as a source, being excluded in boundary conditions. The boundary conditions of other solutions given in the following paragraphs can be determined similarly; each solution corresponds to its own boundary conditions.

The physical description of the solution with constants $c_3 < 0$, $c_7 > 0$ and $c_6 = 0$ is shown in Figs. 1 and 2. The first one presents the flow between two infinite porous plates parallel to x abscissa moving along the abscissa direction with different speeds, which are given by Eq. (27) with $y = 0$ and $y = 1$ to satisfy the non-slip condition in the viscous flow. The flow field between the porous plates is a non-synergy field and described by Eqs. (27)–(29), the x-direction speed u distributes with 1-D exponential function of y, the y-direction speed υ is a constant c_3 in the whole field including in the porous plates. The distribution of heat source q is similar to u; when $c_3 = -c_7\frac{\nu}{a}$, the curves of q completely coincides with that of u. The temperature distribution is a 2-D function: linear along x-direction and exponential in y-direction. The isothermals have to completely identical to the streamlines, the latter can be derived by $dy/dx = \upsilon/u$ and the result is $x = \{(\nu/c_3)^3 \exp[c_3(y+c_6)/\nu] - c_8\}/c_3$. Both streamlines and isothermals in this considered field are shown in Fig. 2. The heat source distribution is a 1-D exponential function of y.

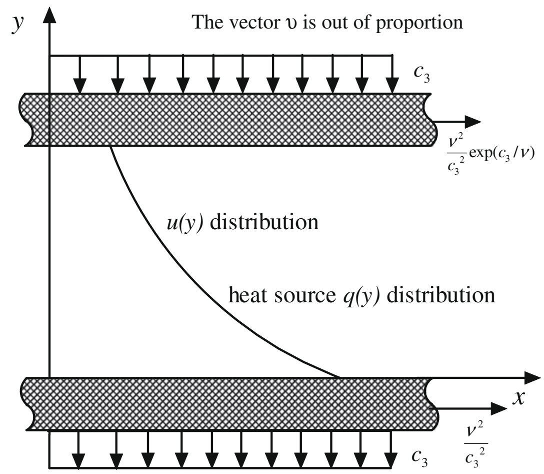

Fig. 1. The distribution of u velocity and heat source of Eqs. (27)–(31) with $c_3 < 0$, $c_6 < 0$, and $c_3 = -c_7\frac{\nu}{a}$.

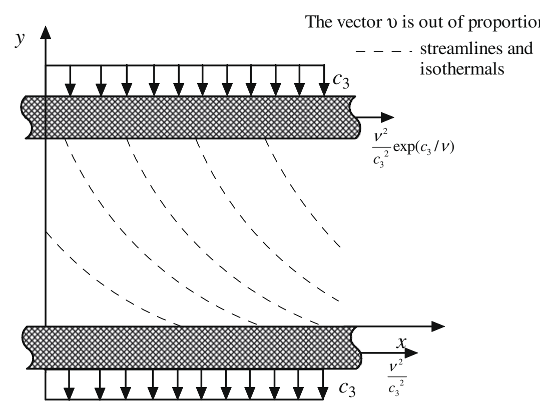

Fig. 2. The streamlines and isothermals of Eqs. (27)–(31) with $c_3 < 0$, $c_6 = 0$, and $c_3 = -c_7\frac{\nu}{a}$.

4. Analytical full field non-synergy solutions with heat source (II) – concise solution family using the method of separating variables with addition

The physical feature of the solution in previous paragraph is a little bit complicated (with two parallel moving porous walls as boundary). In this paragraph, a very simple solution with two infinite parallel steady solid walls as boundary is given.

If the constants c_1, c_3 and c_4 in Eqs. (15)–(18) are equal to zero, then it can be concluded that $\upsilon = 0$ and u is a quadratic function of y. Then the velocity field can express as:

$$u = c_1 y^2 + c_2, \quad (32)$$

$$\text{and} \quad \upsilon = 0. \quad (33)$$

It corresponds to the following assumptions

$$X_u = c_2, \tag{34}$$
$$Y_u = c_1 y^2, \tag{35}$$
$$X_v = Y_v = 0, \tag{36}$$

in Eqs. (6)–(14), and satisfies Eq. (10) with $G = 0$.

Under such assumptions, it is easy to obtain following result from momentum equations Eqs. (11) and (12):

$$X_p = 2c_1 \rho v x, \tag{37}$$
$$Y_p = p_o. \tag{38}$$

Then the pressure formula is

$$p = p_o + 2c_1 \rho v x. \tag{39}$$

Since $\upsilon = 0$, then from Eq. (14), $X_\theta' = 0$ and X_θ is a constant. Using the same equation, it is concluded that Y_θ can be an arbitrary function of y, it means

$$\theta = \text{arbitrary } f(y). \tag{40}$$

At last, the formula of heat source q (Eq. (13)) can be easily solved as

$$q = -af''(y) = -a\theta''(y). \tag{41}$$

Eqs. (32), (33), (39)–(41) represent a family of simple field synergy solutions and which number is infinite since there is an arbitrary function $f(y)$. However, the velocity distribution – parabolic curve along y-direction is the same for all solution family, similar to the classical 2-D Poiseuille flow; the pressure distribution along x-direction is also similar to the Poiseuille flow. The main distinguishing feature is the heat source distribution, it controls the distributions of the temperature and guarantees the field non-synergy, and its function has an evident relationship to the temperature distribution [Eqs. (40) and (41)]. Among the variables, flow velocity u, temperature θ and heat resource q are functions of y, and pressure p is function of x.

Now, we point out some representative functions of $f(y)$ and their effects [the velocities and pressure maintain their expressions as Eqs. (32), (33), (39)].

Of course, we only give some simplest but perhaps useful examples from infinite cases.

4.1. Solution with f(y) = Const

If $f(y)$ = Const., then θ = Const. and $q = 0$, no heat transfer occurs. The solution is approximate to the Poiseuille flow, which is meaningless to the field non-synergy principle.

4.2. Solution with linear temperature distribution

If $f(y)$ is a linear function, for example $f(y) = c_3 y + c_4$, then

$$\theta = c_3 y + c_4, \tag{42}$$
$$\text{and} \quad q = 0. \tag{43}$$

Actually, it is the common Poiseuille flow with linear temperature distribution along y direction. Indeed, such case has been mentioned by Tao et al. [5] as the example of the non-synergy. The simple physical feature of this solution is shown in Fig. 3. By the way, it is worth mentioning a special character of this non-synergy case: the non-synergy condition can be obtained without any external sources; it is unable for the full field synergy condition [10]. Perhaps, it is the best non-synergy (adiabatic) approach to lower the total heat dissipation in a laminar flow region. Actually, there are heat injection along upper wall and heat ejection along lower wall by conduction with equal amount of heat, and no heat source. However, there is no convection, i.e., the non-synergy.

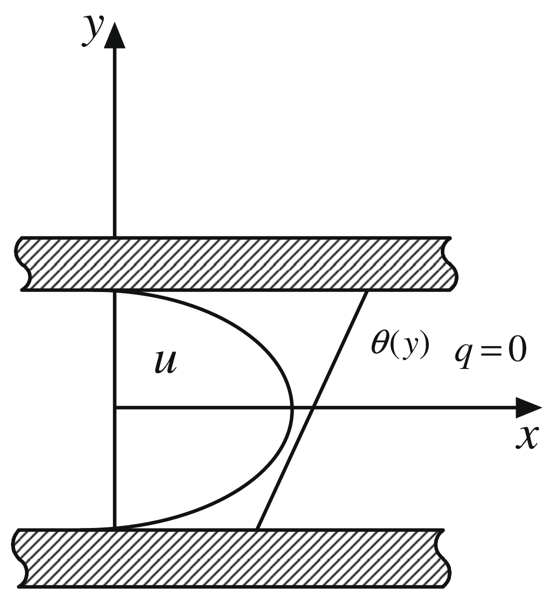

Fig. 3. The physical feature of Eqs. (32), (33), (42), (43).

4.3. Solution with even heat resource

For such case, $q = q_o$ = const., then from Eq. (41), it is obtained:

$$\theta = -q_o y^2/2a + c_3 y + c_4. \tag{44}$$

The temperature is a quadratic function of y. A schematic figure of this solution is given in Fig. 4. Similar to previous sub-paragraph, there is no convection, i.e., non-synergy, which is different from the solution in previous sub-paragraph, there is heat ejection or injection with different values of constants along both upper and lower walls by conduction, but there is even heat source distributed around all flow regions to offset the heat ejection. The total effect is the non-synergy also but with conduction.

4.4. Solutions with similar θ and q distribution

If the $f(y)$ in Eq. (40) is $c_3 \sin(c_4 y) + \theta_o$, we can obtain:

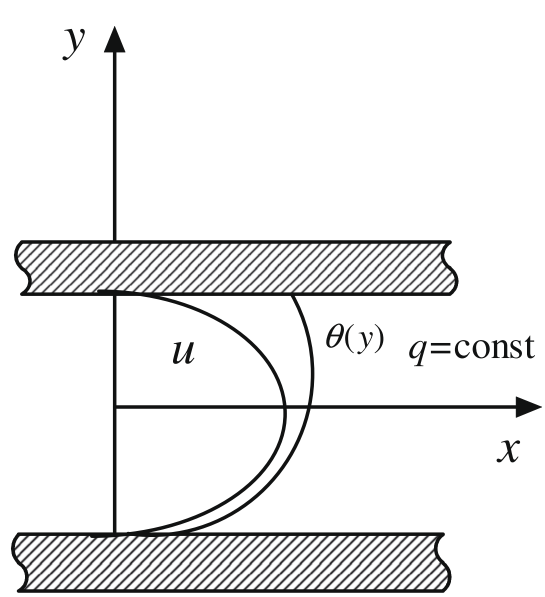

Fig. 4. The physical feature of Eqs. (32), (33), (44).

$$\theta - \theta_o = c_3 \sin(c_4 y), \tag{45}$$

and $$q = ac_4^2 \cdot c_3 \sin(c_4 y) = ac_4^2(\theta - \theta_0). \tag{46}$$

The distributions of temperature and heat resource are similar; the distribution of θ for this solution is roughly similar to Fig. 4. We omitted here to shorten the space.

Another solution example is given as:

$$\theta - \theta_o = c_3 \sinh(c_4 y) \tag{47}$$

and $$q = -ac_4^2 \cdot c_3 \sinh(c_4 y) = -ac_4^2(\theta - \theta_0). \tag{48}$$

The curves of temperature and heat source are presented in Fig. 5 with $c_4^2 = 1/a$. The main difference between Eq. (46) and (48) is that in the former case heat is ejected but the latter is injected. Perhaps the heat injection is easier to accurately set in practice than the heat ejection.

4.5. Other possible solutions

Using Eqs. (32), (33), (39)–(41) and choosing different $\theta = f(y)$, infinite number of field synergy solutions can be derived easily, their velocity and pressure distributions are similar to the Poiseuille flow, only the temperature distribution $\theta(y)$ and corresponding heat source distribution are different. By the way, we have not yet found some other solutions, which have physical character more clear than that in Figs. 3–5. However, perhaps new solutions could be found later from the practical application. At least, the solutions given in this paragraph are based on the Poiseuille flow, it is a very popular case.

5. Analytical full field non-synergy solution with heat source (III) – using the hybrid method of separating variables

Indeed, the method of separating variables with addition have been successfully applied to derive many analytical solutions, however, for simultaneous equations with two or more variables, it is not necessary to apply the same separating approach for different variables. For example, some variables are treated with the method of separating variables with addition, and the others are treated with the common method of separating variables with multiplication. In this paragraph, it is assumed that

$$\theta(x, y) = X_\theta \cdot Y_\theta. \tag{49}$$

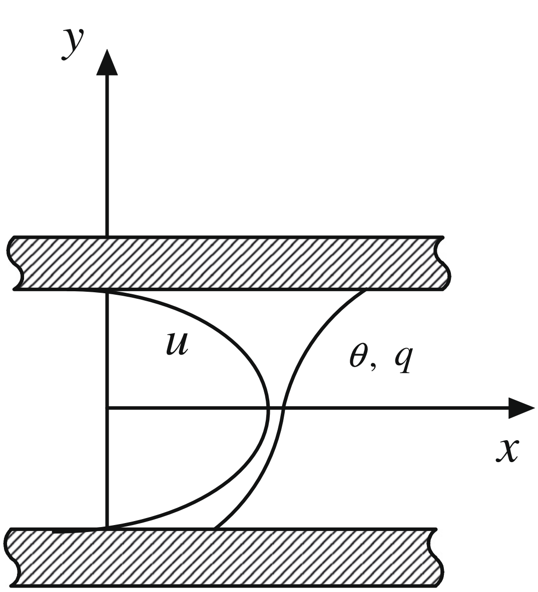

Fig. 5. The physical feature of Eqs. (32), (33), (47), (48).

instead of Eq. (9), but other variables are assumed as the same as before Eqs. (6)–(8).

In this case, the Eqs. (10)–(12) are also effective, but the Eq.(14) has to be changed to

$$(X_u + Y_u)X'_\theta Y_\theta = -(X_\upsilon + Y_\upsilon)X_\theta Y'_\theta, \tag{50}$$

then the velocity and pressure distributions may as the same as Eqs. (27)–(29). But the temperature distribution should be derived by Eq. 50 with known velocity distribution Eqs. (27) and (28). The expression is

$$(\nu/c_3)^2 \exp[c_3(y + c_6)/\nu]X'_\theta Y_\theta = -c_3 X_\theta Y'_\theta. \tag{51}$$

After separating variables, following two ordinary differential equations are obtained:

$$-X'_\theta/X_\theta = c_7 = Y'_\theta/Y_\theta \frac{1}{(\nu^2/c_3^3)\exp[c_3(y + c_6)/\nu]}. \tag{52}$$

The final result of Eq. (52) is

$$\theta = \theta_0 + c_8 \exp(-c_7 x) \cdot \exp\{c_7 c_3{}^2 \exp[c_3(y + c_6)/\nu]/\nu\}. \tag{53}$$

The heat source q can be derived easily by substituting the expressions of u, υ and θ [Eqs. (23), (24), (53)] into Eq. (4).

The physical feature of this solution is very similar to that presented by Figs. 1 and 2 given in paragraph 3, only the heat source q here is a 2-D function. The graphical expressions are not given here to shorten the space of this paper.

Besides obtaining an exact solution, another more meaningful result of this paragraph is the effectiveness of new method of separating variables for partial differential equations. It has to be developed further.

6. Analytical full field non-synergy solutions with mass flow source – using the method of separating variables with addition

All the solutions given in abovementioned three paragraphs only apply the heat source to arrive field synergy. In this paragraph, solutions utilizing mass flow sources only to arrive field non-synergy are derived. However, utilizing pure mass flow sources is commonly more complicated in practice compared to heat sources. For example, the temperature of each particle of the mass source has to be the same as the temperature at the injecting particle positions, otherwise it is difficult to accurately control the temperature field (there is no problem for ejection); in addition, the particle motion would commonly disturb the flow field also. Nevertheless, deriving some analytical solution is helpful for field non-synergy principle and how to promote the field non-synergy.

Different from the above paragraphs, in the following derivation, $q = 0$ and $G \neq 0$ is adopted in the governing equation set.

Omitting the trial and error procedure, the brief derivation procedure is summarized as:

Let $$Y_u = \text{Const.} = c_1, \tag{54}$$

$$X_\upsilon = \text{Const.} = c_2. \tag{55}$$

Then, the governing Eqs. (11)–(14)) can be rewritten as:

$$(X_u + c_1)X'_u = -\frac{1}{\rho}X'_p + \nu X''_u, \tag{56}$$

$$(Y_\upsilon + c_2)Y'_\upsilon = -\frac{1}{\rho}Y'_p + \nu Y''_\upsilon, \tag{57}$$

$$(X_u + c_1)X'_\theta + (Y_\upsilon + c_2)Y'_\theta = a(X''_\theta + Y''_\theta) \tag{58}$$

and $$(X_u + c_1)/(Y_\upsilon + c_2) = -Y'_\theta/X'_\theta \tag{59}$$

After separating variables, we can obtain the following expressions:

$$X'_p = \rho\left[-(X_u + c_1)X'_u + \nu X''_u\right], \quad (60)$$
$$Y'_p = \rho\left[-(Y_\upsilon + c_2)Y'_\upsilon + \nu Y''_u\right], \quad (61)$$
$$aX''_\theta - (X_u + c_1)X'_\theta = -c_3 = (Y_\upsilon + c_2)Y'_\theta - aY''_\theta \quad (62)$$
$$\text{and} \quad (X_u + c_1)X'_\theta = c_4 = -(Y_\upsilon + c_2)Y'_\theta \quad (63)$$

From Eqs. (62), (63), following results can be deduced:

$$\theta = (c_4 - c_3)x^2/(2a) + c_6 x + c_7 + (-c_4 + c_3)y^2/(2a) + c_8 y. \quad (64)$$

Then the velocities can be derived from Eq. (59) as:

$$u = c_4 a/[(c_4 - c_3)x + c_6 a], \quad (65)$$
$$\upsilon = -c_4 a/[(-c_4 + c_3)y + c_8 a]; \quad (66)$$

and from Eqs. (60), (61) we can obtain the pressure expression:

$$p = p_0 - c_4 \rho a\Big\{[c_4 a + 2\nu(c_4 - c_3)]/[(c_4 - c_3)x + c_6 a]^2 + \left[-c_4 a + 2\nu(-c_4 + c_3)\right]/[(-c_4 + c_3)y + c_8 a]^2\Big\}/2 \quad (67)$$

In addition, the mass source G, which is derived from Eq. (1), can be expressed as:

$$G = -c_4 a\Big\{(c_4 - c_3)/[(c_4 - c_3)x + c_6 a]^2 + (c_4 - c_3)/[(-c_4 + c_3)y + c_8 a]^2\Big\} \quad (68)$$

If $c_4 \geqslant c_3$ and $c_4 > 0$, then $G < 0$, the mass flow source is negative, the mass ejects from the system; as mentioned before, this condition is easier to be achieved in practice.

However, no evidently physical feature of such solutions has been found, this solution perhaps can only be a benchmark solution for the non-synergy CHT.

The Eqs. (64)–(68) are rather complicated. If assuming $c_3 = c_4$ and $c_6 = c_8 = 1$, the equation set can be simplified as:

$$u = c_4 = \text{const.} \quad (69)$$
$$\upsilon = c_4 = \text{const.} \quad (70)$$
$$P = p_0 = \text{const.} \quad (71)$$
$$\theta = x + y + c_7 = \theta(x, y) \quad (72)$$
$$G = 0 \quad (73)$$

The streamlines (and isothermals) can be introduced by

$$\frac{dy}{dx} = \frac{\upsilon}{u} \quad (74)$$

The final result is

$$y = -x + c_9. \quad (75)$$

The physical feature of the simplified solution Eqs. (69)–(73) is presented in Fig. 6. In fact, it is a parallel flow between two parallel infinite long solid walls, an extremely simple case. If the coordinates are anticlockwise rotated by 45°, it is similar to the former case, but the distributions of velocity and temperature are different.

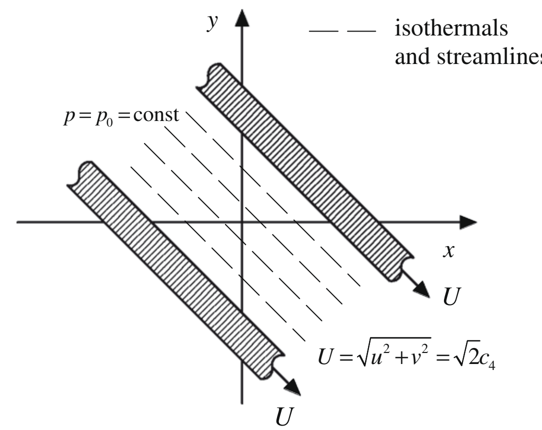

Fig. 6. The physical feature of simplified solution Eqs. (69)–(73), (75).

7. Summary

1. The field synergy principle – an approach to find out best convection condition – has been successfully proposed, demonstrated and applied, it is worth studying the opposite research direction – the non-synergy – to find out worst convection condition.
2. A basic research on the non-synergy is accomplished: several algebraically explicit analytical exact solutions are derived for steady 2-D incompressible laminar flow. Such solutions have their own theoretical meaning and can be the strict benchmark solutions to promote the computational heat transfer. According to the knowledge of authors, they are the only analytical exact solutions for the non-synergy.
3. The derived solutions include conditions with heat source, mass flow source and without any source, some physical feature of the solutions are discussed and explained by figures.
4. For the very complicated partial differential equation set of the non-synergy, the derivation method mainly adopts some extraordinary approaches successfully applied by the authors recently, such as the method of separating variables with addition and hybrid method. The derivation results show again that these methods are very effective.

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